1970

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Williamson, T. J., and R. W. Albrecht. "Calculations of Neutron Time-Energy Distributions in Heavy Moderators." Nuclear Science and Engineering 42, no. 1 (1970): 97-100.

http://hdl.handle.net/10945/57635

Far enough from the source, the magnitude of its spatial shape will be small and smoothly varying. Therefore, either $F_m(\bar{r})$ is also smoothly varying or, if it is not, c_m is small. Under these conditions the magnitudes of the $\psi_m(\bar{r})$ will change only slowly with $\bar{r}$ and thus, far from the source, G can be expected to level off and finally reach a constant value.

VI. CONCLUSIONS

When the β/Λ parameter yielded by the Garelis-Russell technique is spatially dependent the correct β/Λ_1 can in principle be calculated [Eq. (35)] when the parameter is obtained at positions throughout the assembly. Existing spatial symmetry in the experiment should be used to reduce the requirements.

The attractiveness of the Garelis and Russell (β/Λ)—method is mainly due to the fact that it "integrates" over the modes and permits an experimental determination of (β/Λ). Thus the use of a correction [Eq. (35)], where explicit modal dependence has to be calculated from a theoretical model, somewhat defeats the simplicity and usefulness of the method.

Calculations of Neutron Time-Energy Distributions in Heavy Moderators

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Received April 24, 1970

The task of making detailed neutronics calculations for heavy moderating isotopes has always been a most difficult problem for a variety of reasons. Consider, for instance, the problem of calculating the time and energy (or lethargy) dependent neutron density $N(E,t)$ that develops in a homogeneous moderator during and following the introduction of a source, $S(E,t)$ in the MeV range. Monte Carlo techniques are very impractical because of the large number of collisions required per history. The usual multigroup stepping matrix techniques[1] are inapplicable for at least two reasons: (a) inordinately large matrices are required when collision energy losses are small and (b) scattering through the inelastic region may take but a microsecond while moderation to thermal energies may require times of the order of a millisecond, thus making it virtually impossible to choose a suitable time step upon which to define the matrix.

Analytical techniques[2,3] have been reasonably successful in predicting the low energy or asymptotic shape of $N(E,t)$ but they are incapable of dealing with the complexity of inelastic scattering, the effects of which carry over to relatively low energies.

We have been able to overcome many of these difficulties using our recently developed discrete stochastic model.[4] The model is similar in many respects to the usual time-dependent multigroup theory in which a multigroup neutron spectrum, represented here by a state vector $\bar{s}(t)$

$$\bar{s}(t) = [s_1(t), s_2(t), \ldots,] \tag{1}$$

is stepped forward in time by repeated multiplication into a stepping matrix $\bar{\bar{P}}_{\Delta t}$

$$\bar{s}(t + \Delta t) = \bar{s}(t) \cdot \bar{\bar{P}}_{\Delta t} \quad , \tag{2}$$

where the elements $P_{i,j}(\Delta t)$ of $\bar{\bar{P}}_{\Delta t}$ are probabilities for transition between groups i and j during the interval Δt, and $s_i(t)$ is the population of group i. The new model differs from multigroup theory in the method of calculating transition probabilities and in the use of a well-known result of the theory of Markov processes which allows the stepping interval to be conveniently increased

$$\bar{\bar{P}}_{n\Delta t} = (\bar{\bar{P}}_{\Delta t})^n \quad . \tag{3}$$

Equation (3) provides a mechanism for the generation of a stepping matrix with transition probabilities of consistent accuracy for any step and energy range.

A requirement for maximum accuracy in multigroup calculations of elastic scattering processes is that the groups be small enough to allow neutrons to scatter from the mean scattering energy in one group to the smallest energy of the next lower group. For moderating isotopes with mass $A \sim 200$ this requires at least 160 groups per decade in energy. Unfortunately, on most computers a stepping matrix of dimension 200×200 represents a practical upper limit (both in terms of computation time and storage requirements) while typical slowing down problems span four or more decades.

We have circumvented this restriction by allowing the stepping matrix to cover only that region of energy occupied by the neutron population at a given instant. It is a well-known result of collision theory[5] that with purely elastic scattering the asymptotic energy distribution of moderating neutrons is nearly Gaussian with a dispersion (relative standard deviation) no larger than

$$D = \left[\frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle^2}\right]^{1/2} = \left(\frac{8}{3A_L}\right)^{1/2} \quad , \tag{4}$$

where A_L is the atomic mass of the lightest isotope present. As the neutron population moves downward in energy following a short burst from the source, we allow the matrix to move with it using the previously mentioned technique to generate new rows of transition probabilities at the bottom of the matrix, and to increase time steps as appropriate (Fig. 1). In the calculation described below, a real matrix of dimension 200×200 was carried down the diagonal of a "virtual" (elements not stored in computer) matrix of dimension 700×700.

To reduce calculation times we have used the fact that in the elastic scattering region, transition probabilities decrease geometrically along a row moving away from the diagonal. As a result, for almost any useful time step and any moderator with $A > 10$, the transition probability

[1] A. K. GHATAK and H. C. HONECK, *Nucl. Sci. Eng.*, **21**, 227 (1965).

[2] R. E. MARSHAK, *Rev. Mod. Phys.*, **19**, 185 (1947).

[3] A. A. BERGMAN et al., *Proc. First Intern. Conf. Peaceful Uses At. Energy*, **4**, 135 (1955).

[4] T. J. WILLIAMSON and R. W. ALBRECHT, *Nucl. Sci. Eng.*, **37**, 41 (1969).

[5] K. H. BECKURTS and K. WIRTZ, *Neutron Physics*, p. 174, Springer-Verlag, New York (1964).

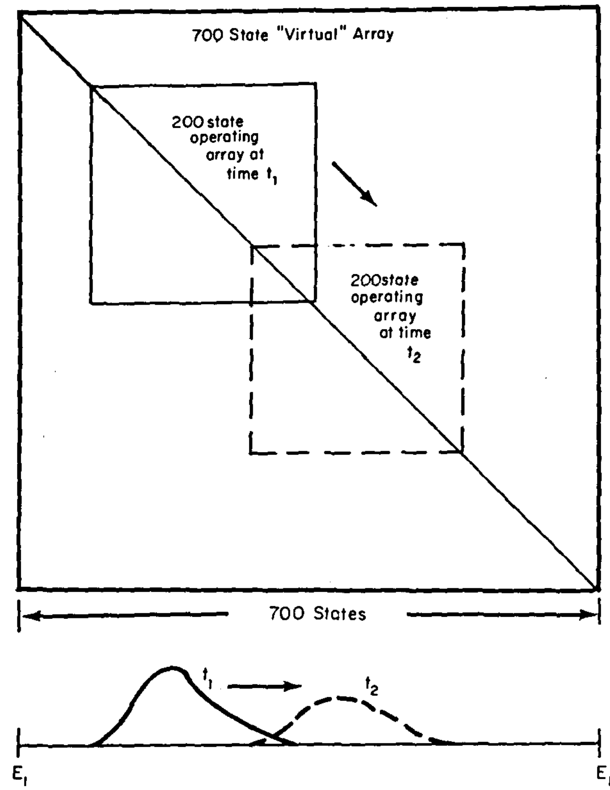

Fig. 1. Scheme for allowing transition matrix to follow neutron population.

$P_{i,j}(\Delta t)$ becomes vanishingly small (say $< 10^{-6}$) if $j > i + 10$. Thus, when calculations are being performed it is unnecessary to include transitions to a state j from any state i that is more than 10 states above j.

Application of our model does require two important compromises. The lesser of the two is the assumption of a normal mode space dependence. This is somewhat justified by the long mean-free-paths characteristic of heavy moderators. The more important compromise is made necessary because of the cooperative effect of inelastic scattering and time distributed sources in dispersing the neutron distribution. Inelastic scattering alone will cause neutrons from a 14 MeV source to be spread out to below 100 keV within a few nanoseconds. Once they are below the inelastic threshold the neutrons lose energy far more slowly and the narrow asymptotic shape is developed after a few collisions. However, because of the initial dispersal of neutrons over the broad inelastic region it is impossible to provide a matrix large enough to treat elastic scattering properly. Instead a few relatively broad groups must be used to span the energy region in which inelastic scattering reactions are the dominant moderating process. Within this region the elastic scattering kernel is adjusted by a scaling process that has been described elsewhere.[4] We have found that scaling tends to artificially disperse the energy distribution without significantly altering the mean energy. Since the inelastic kernel overwhelmingly dominates the shape of the distribution, scaling of elastic cross sections introduces only very minor errors.

As an example of the application of the model to heavy moderators we have chosen to study the lead slowing down spectrometer. Such devices have been in use for several years, in Russia,[3] in Germany,[6,7] and elsewhere. Lead spectrometers usually have a simple configuration such as a large lead cube resting on a concrete pedestal and penetrated by a 14 MeV pulsed neutron source and a small sample channel. If the source pulses are of short duration the neutrons will eventually develop a relatively narrow distribution in energy and the mean energy of the distribution can be easily related to elapsed time. By observing the time dependence of the emission of capture gammas from the sample being studied one can deduce the energy dependence of cross sections. Popov and others have made a number of studies of (n,γ) cross sections by this means.[8-11] More recently, there has been active study of temperature effects on the energy dependence of (n,γ) cross sections of various fast reactor materials.[7] Such studies are considered a necessary adjunct to the analysis of integral Doppler experiments and the theoretical analyses of uncertainties in nuclear data.

Two physical properties of a given spectrometer must be known before it is possible to extract cross sections from reaction rates and determine the accuracy of the estimates.

First it is necessary to know the mean energy of the neutrons as a function of the time elapsed from initiation of the neutron burst. It is easy to show that with a source that is a δ-function in time and velocity, and with a purely scattering medium of atomic mass A and scattering cross section Σ_s, the mean energy $\langle E(t)\rangle$ is given by[3,7]

$$\langle E(t)\rangle = \frac{M_N A^2}{2\Sigma_s^2}\,\frac{1}{\left(t + \dfrac{A}{v_0\Sigma_s}\right)^2} \quad , \tag{5}$$

where v_0 is the source neutron velocity and M_N is the neutron mass. Although the above result is based on a highly idealized model, our calculations have shown that it works very well over the entire energy range below the inelastic threshold. With realistic source burst widths of up to 4 μsec the formula has been found to be satisfactory if t is replaced by elapsed time from mean emission time.

A far more difficult problem is posed by the desire to know something about the energy or lethargy distribution of the neutrons as time elapses. For most purposes it is sufficient to determine the relative standard deviation in energy (also commonly referred to as dispersion, or resolution) of $N(E,t)$ as a function of either the mean energy or the elapsed time. Clearly this parameter will determine the energy resolution of measured cross sections. Seufert and Stegemann[7] have concluded that for their Doppler measurements it is necessary that

$$\frac{\Gamma_r}{\langle E\rangle} \ll \frac{\overline{D^s}}{\langle E\rangle} \ll D \ll 1 \quad , \tag{6}$$

where $\overline{D^s}$ is the mean level spacing, Γ_r is the gamma width of the resonances, and D is the dispersion of the neutrons [Eq. (4)].

[6] H. SEUFERT, "Untersuchung des Dopplereffektes in schnellen Neutronenspektren nach neuen experimentellen Methoden," Dissertation, Karlsruhe (1968).

[7] H. SEUFERT and D. STEGEMANN, Institute für Neutronenphysik und Reaktortechnik, KFK-631, Kernforschungszentrum, Karlsruhe (1967).

[8] A. I. ISAKOV, YU. P. POPOV, and F. L. SHAPIRO, *J. Exptl. Theor. Phys.*, (USSR), **38**, 989 (1960).

[9] N. T. KASHUKEEV, YU. P. POPOV, and F. L. SHAPIRO, *J. Nucl. Energy*, **14**, 76 (1961).

[10] YU. P. POPOV and F. L. SHAPIRO. *Sov. Phys. JETP*, **15**, 683 (1962).

[11] S. A. ROMANOV and F. L. SHAPIRO, *Sov. J. Nucl. Phys.*, **1**, *2*, 159 (1965).

Our calculations have employed the 26-group ABN cross-section set[12] with an additional group added to cover the region from 10.5 to 14 MeV. The few resonances that occur in lead were ignored since they will not affect dispersion calculations significantly. The broad group cross sections were distributed over a fine group structure consisting of about 14 groups per decade above 300 keV and about 159 groups per decade below 300 keV. A geometric buckling of B^2 = 0.000558 corresponds to the physical dimensions of a large spectrometer (2.3 m on a side) but ignores such complications as the albedo of the concrete pedestal it sits on.

Output from the calculations consisted of (a) the neutron density in lethargy $N(u,t)$ and the relative standard deviation in energy and velocity at selected times following the initiation of the source, (b) time-dependent capture and leakage rates, and (c) steady-state flux.

The calculation was terminated when the neutrons reached 20 eV, at which point 75.7% had been lost to leakage, 12.2% had been captured in the lead and 12.1% were still slowing down.

Figure 2 displays the calculated lethargy-dependent neutron densities following source pulses of 0, 1, 2, and 4 μsec duration. Certain features of these figures are worth noting:

1. In Fig. 2a the structure and boundaries of the cross-section set are quite apparent. At 2 and 8 nsec the spectrum is almost completely determined by inelastic transfer from the source group.

2. In Figs. 2c and 2d a steady-state situation exists above 200 keV at the time the source is turned off.

3. A striking feature in all four cases is the rapidity with which a Gaussian shape is achieved by the population below 100 keV.

4. An estimate of the relative widths of the distributions at a given time is provided by the heights of the curves. Since the areas under the curves should be about the same at a given time (presuming the source is off) the width of the curve, or equivalently D, will vary inversely as the height. Thus at 50 μsec the distribution from a 4 μsec source is about 20% broader than that from a 1 μsec source.

A more complete description of the resolution of the pulses as a function of energy and elapsed time is given in Fig. 3. Here the abscissa is actually the ratio of D to its asymptotic value.

On the same figure we have plotted for comparison the

[12] L. P. ABAGYAN, N. O. BAZAZYANTS, I. I. BONDARENKO, and M. N. NIKOLAEV, *Group Constants for Nuclear Reactor Calculations*, Consultants Bureau, New York (1964).

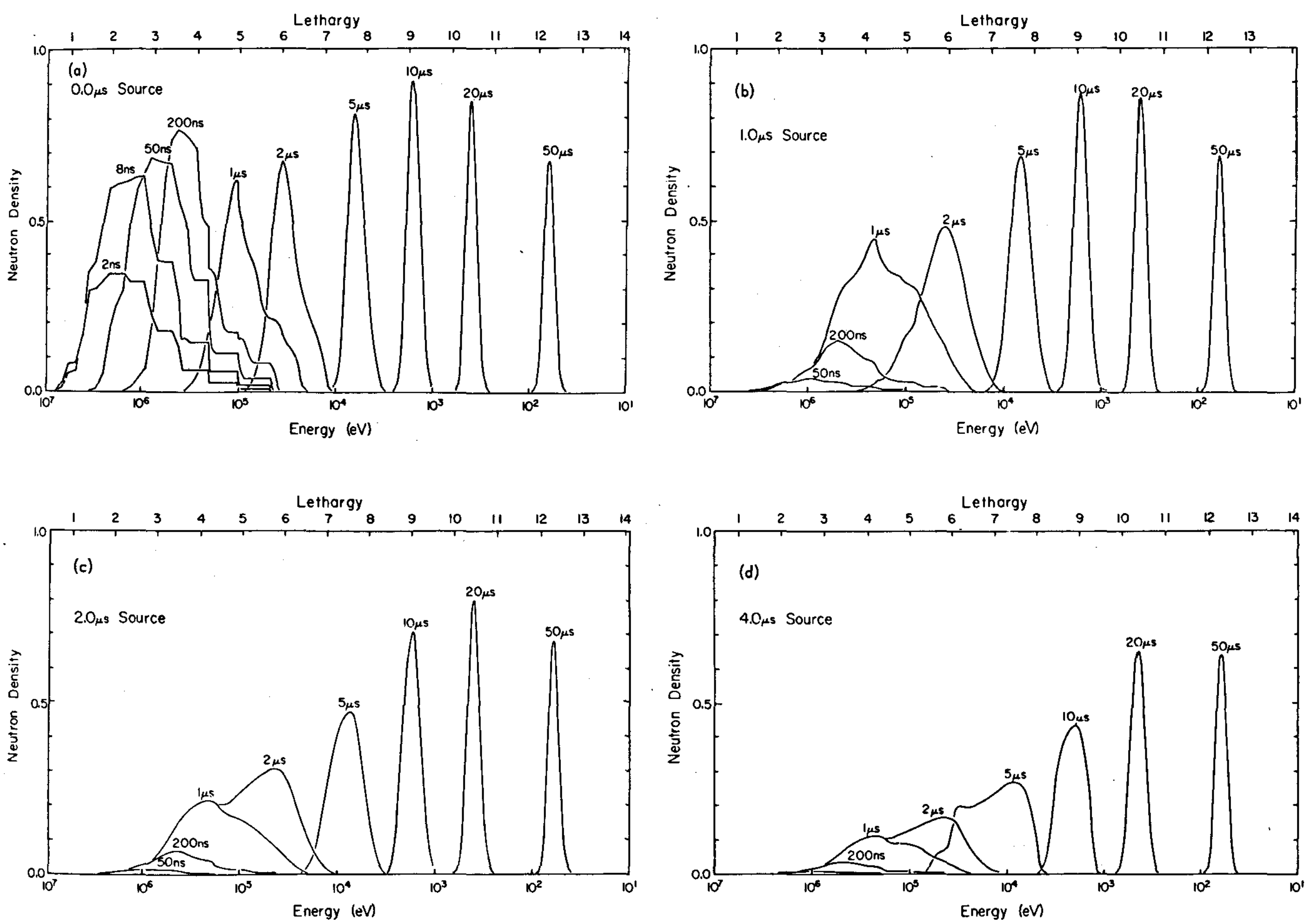

Fig. 2. The function $N(u,t)$ for source burst widths of 0, 1, 2, and 4 microseconds. Normalized to one source neutron at 14 MeV.

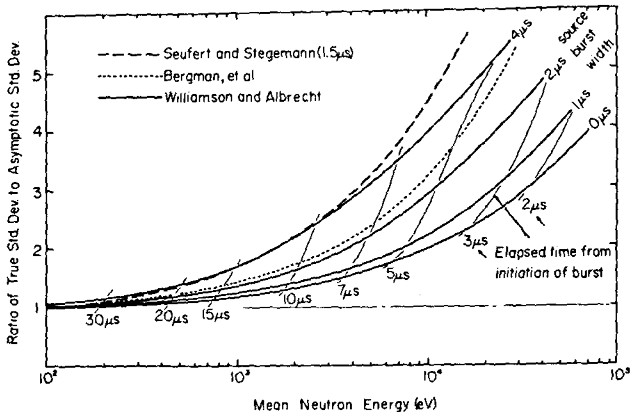

Fig. 3. Ratio of true to asymptotic value of relative standard deviation of $N(E,t)$ as a function of source burst width, mean energy, and time elapsed from initiation of source burst.

prediction given by Bergmann et al.[3] (source time unspecified) and by Seufert for a 1.5 μsec source. Our results clearly provide a more optimistic estimate of the resolution of the spectrometer. Seufert's results are based on a model first proposed by Bergmann who suggested that the relative standard deviation in velocity ought to obey the following equation below 100 keV[a]:

$$\left[\frac{\langle v^2\rangle - \langle v\rangle^2}{\langle v\rangle^2}\right]^{1/2} = \left[\frac{2}{3A}\left(1-\frac{\langle E\rangle}{E_0}\right) + D_0\frac{\langle E\rangle}{E_0}\right]^{1/2} , \qquad (7)$$

where $\langle E\rangle$ is the mean energy, E_0 = 100 keV and the constant D_0 was estimated to be ~0.3. The theoretical basis for this expression is obscure. In fact, in light of recently developed expressions for the lethargy dependence of velocity moments we would expect a better formulation to be a power series in $\left[\frac{\langle E\rangle}{E_0}\right]^{1/2}$.[13] As a useful approximation however, the following expression provides results that compare well with calculations:

$$\left[\frac{\langle v^2\rangle - \langle v\rangle^2}{\langle v\rangle^2}\right]^{1/2} = \left[\frac{2}{3A} + D_0\left(\frac{\langle E\rangle}{E_0}\right)^{0.84}\right]^{1/2} , \qquad (8)$$

where, again, E_0 = 100 keV, but D_0 is ~0.07 for source widths of less than 1 μsec. The much smaller value of D_0 is a consequence of the very rapid focusing of the distribution below the inelastic threshold, a fact that is certainly not intuitively obvious.

In practical applications of the spectrometer one of the overriding factors is the existence of background radiation, mainly from capture gammas produced in the lead. In Fig. 4 the specific capture rate following 0 and 4 μsec bursts has been plotted. Consistent with reports of experimenters, a high background should persist for up to 5 μsec following the burst.

[a] Our numerical calculations have shown that the following relation seems to hold true at all energies:

$$\frac{\dfrac{<v^2> - <v>^2}{<v>^2}}{\dfrac{2}{3A}} = \frac{\dfrac{<E^2> - <E>^2}{<E>^2}}{\dfrac{8}{3A}} .$$

The two quantities differ by no more than 5% above 100 keV and by less than 1% below 100 keV. So far, we have not obtained analytical verification of the result.

[13] T. J. WILLIAMSON, *Nucl. Sci. Eng.*, **39**, 273 (1970).

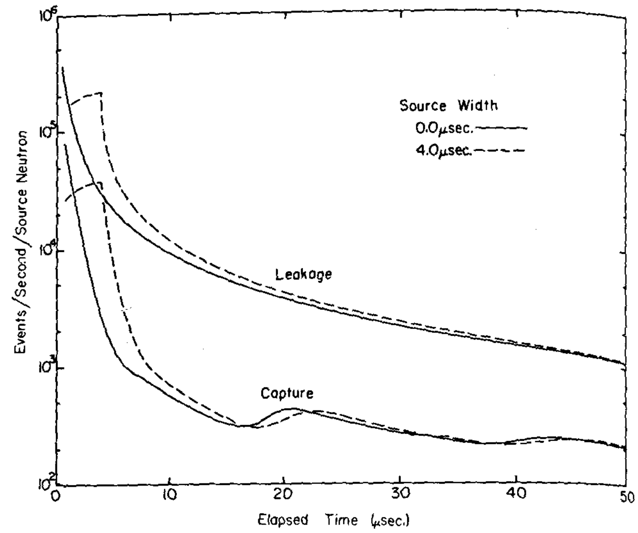

Fig. 4. Time dependence of leakage and capture for 0 and 4 μsec sources.

In closing, it should be noted that certain errors are inherent in the calculation and these are summarized below.

In a previous paper[4] it was shown that generally multigroup calculated dispersions will be larger than the true values. With the optimized stochastic model D is about 5% too large at all energies; however, this error cancels out almost completely in the ratios plotted in Fig. 3.

The leakage rates in Fig. 4 are probably pessimistically large early in the history of the pulse. This is because neutrons are actually born near the center of the cube rather than in a fundamental mode. Clearly a neutron born at the center has a smaller probability for leakage during its lifetime than one born elsewhere. Because of the overestimate of leakage we would expect that the distribution in Fig. 2 should properly have slightly greater amplitude.

The Formulation of Continuous Slowing Down Theory for General Processes in Terms of Separable Kernels

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Received February 9, 1970

Recently, there has been considerable interest in the application of continuous slowing down theory to problems in Fast Reactor Analysis.[1-4] One difficulty encountered in such applications is the manner in which inelastic scattering has been treated. Driscoll and Kaplan[1] defined the

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[2] F. E. DUNN and M. BECKER, *Trans. Am. Nucl. Soc.*, **11**, 213 (1968).

[3] F. E. DUNN and M. BECKER, *Trans. Am. Nucl. Soc.*, **12**, 640 (1969).

[4] M. SEGEV, *Trans. Am. Nucl. Soc.*, **12**, 640 (1969).